A Mereology for Emergence

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Abstract

The paper first investigates the tension between reductive accounts of mereological structure and emergence as characterized in Jessica Wilson's seminal work. It then suggests a new mereology for emergence. Finally, the resulting account is applied to a paradigmatic case of an emergent whole.

Keywords: Emergence, Mereological structure, Mereological sum, Matter.

To my partner in crime, J.W.

1. Emergence and Mereological Reductionism

There are several broadly "reductive" accounts of mereological structure. They all try to capture rigorously the somewhat vague intuition that "wholes are nothing over and above their parts". The most radical view in the reductive camp holds that mereological composition is strict numerical identity, in that wholes are numerically identical to their parts considered collectively. The view is known as *Strong Composition as Identity*. Using double signs (such as xx), for plural terms:[1]

> *Strong Composition as Identity* (CAI): If the xx compose y, then $xx = y$.

There is a famous argument in the literature against CAI from the possibility of emergence.[2] It goes roughly as follows. If CAI is true, then wholes cannot have properties that the plurality of their proper parts do not have. Emergent properties are exactly an example of such properties. Hence, if (possibly) there is emergence, CAI is false. Whatever one thinks of the argument, CAI is indeed a radical option. For example it might require substantive changes in the logic of identity and/or comprehension principles of plural logic. Hence, it is important to realize that the tension between reductive accounts of mereological structure and (the possibility of) emergence cuts a little deeper. As Wilson (2021) puts it,

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[1] For an introduction see Baxter and Cotnoir 2013.

[2] See e.g., McDaniel 2008, Schaffer 2010, Sider 2013, and Calosi 2016.

It is the *coupling of cotemporal material dependence* with *ontological* and causal *autonomy* which is most *basically definitive of the notion of emergence*, at least as suggested by the central cases of special-science entities with respect to the physical micro-configurations which are their constant companions (Wilson 2021: 1; italics added).

In the light of this, the general threat coming from emergence to reductive accounts of mereological structure is the following. If emergent wholes are *ontologically autonomous* from their (microscopic) constituents,[3] then they are indeed "something over and above" those constituents, contra the spirit, not just the letter, of reductive accounts. It is not my purpose here to respond to the threat, nor to dissect its presuppositions. Rather, it is to take such a threat at face value and propose a new mereological system that vindicates the claim that "wholes are something over and above their parts"—as seems to be required by metaphysical emergence. This is by no means an easy task. Indeed, many think that mereology alone is not enough to account for complex, highly structured, emergent wholes. This is why they recommend different forms of hylomorphism.[4] Others think that we need to revisit the very mereological framework we use, for example adopting a so-called slot-mereology,[5] or rejecting mereological monism, roughly the view that there is only one notion of (mereological) part.[6] I am going to suggest a mereological account that uses only one notion of parthood. In a nutshell, I am going to suggest that we can define a notion of mereological sum that is not equivalent to extant ones in the literature. Given anti-symmetry of parthood, it turns out that sums are unique. I then define the notion of the matter of an entity as *the* sum of its proper parts. This helps me draw a distinction between *Reducible Wholes*, wholes that are nothing over and above their matter, and *Irreducible Wholes*, wholes that are distinct from their matter. Finally, I suggest that if a whole is an emergent whole, then it is an irreducible whole—as previously defined.[7]

2. A New Mereology

There are three notions of mereological sum in extant literature.[8] I will use $<$ for parthood, $\ll$ for proper parthood, $\circ$ for overlap, defined as usual, and $\prec$ for the plural logic relation of "being one of".[9] For the sake of readability "$xx < y$" abbreviates "$\forall x(x \prec xx \rightarrow x < y)$", and "$x \circ xx$" abbreviates $\exists y(y \prec xx \wedge x \circ y)$". Then the usual notions of sum are defined as follows:

D. 1 $Sum_1(xx, y) \equiv \forall x(x \circ y \leftrightarrow x \circ xx)$ SUM$_1$

D. 2 $Sum_2(xx, y) \equiv xx < y \wedge \forall x(x < y \rightarrow x \circ xx)$ SUM$_2$

D. 3 $Sum_3(xx, y) \equiv xx < y \wedge \forall x(xx < x \rightarrow y < x)$ SUM$_3$

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[3] I follow Wilson (2021: 10) here. Roughly, an emergent whole is a whole with an emergent feature.

[4] See e.g., Koslicki 2008, Fine 2010, and Sattig 2015.

[5] See e.g., Bennet 2013 and Sattig 2021.

[6] See e.g., Canavotto and Giordani 2020.

[7] I developed the technical work on the new mereological system together with Alessandro Giordani. See Calosi and Giordani 2023a, and Calosi and Giordani 2023b.

[8] See Cotnoir and Varzi 2021.

[9] That is, $x \ll y \equiv x < y \wedge x \neq y$, and $x \circ y \equiv \exists z(z < x \wedge z < y)$.

In plain English, y is a Sum_1 of the xx iff it overlaps all and only the things that the xx overlap, y is Sum_2 if every xx is part of y and every part of y overlaps the xx, and finally, y is a Sum_3 iff every xx is part of y, and everything that includes the xx includes y. It is well-known that in mereologies that are weaker than classical mereology, the three notions are not equivalent.[10] Do they exhaust the notions of Sum definable in terms of $<$ and $\prec$? Hardly so. Consider the following:

D.4 $Sum(xx, y) \equiv xx < y \land \forall x(\neg xx \circ x \to \neg x \circ y)$
$\quad \land \forall x(xx < x \to y < x)$ $\qquad\qquad$ SUM

Definition **D.4** simply says that y is the Sum of the xx iff (i) the xx are part of y, (ii) whatever is disjoint from the xx is disjoint from y, and (iii) everything that includes the xx includes y. In other words, according to (i), the mereological sum of a plurality should be inclusive enough to count every member of xx as a part. According to (ii), it should be no more inclusive than that. Finally, according to (iii), a mereological sum should be minimal, in that it has to be part of everything that includes the original plurality. It is easily seen that, in the absence of strong mereological principles we have (1) and (2) below, where i ranges over the three notions of sum in **D.1-D.3**:

(1) $Sum(xx,, y) \to Sum_i(xx, y)$
(2) $Sum_i(xx, y) \nrightarrow Sum(xx,, y)$

Thus, Sum is strictly stronger than any Sum_i. Once we have such a stronger notion of Sum, we can put forward an explicit mereological system based on that notion.[11] For the sake of simplicity, I am going to require a very strong principle for the existence of Sum-s. In particular I am going to require a counterpart of the *unrestricted composition* principle of classical mereology.[12] It should be noted however that weaker principles will do as well. I will return to this shortly. Here is the system:

P.1 $x < y \land y < x \to x = y$ $\qquad\qquad$ ANTISYMMETRY
P.2 $x < y \land y < z \to x < z$ $\qquad\qquad$ TRANSITIVITY
P.3 $x \ll y \to \exists w \exists z(w \ll y \land z \ll y \land \neg w \circ z)$ $\quad$ QUASI-SUPPLEMENTATION
P.4 $x \prec xx \to \exists y\big(Sum(xx, y)\big)$ $\qquad\qquad$ UNRESTRICTED SUM

Let us define "being mereologically simple" and being "mereologically composite" as usual:

D.5 $S(x) \equiv \neg \exists y(y \ll x)$ $\qquad\qquad$ SIMPLE
D.6 $C(x) \equiv \neg S(x)$ $\qquad\qquad$ COMPOSITE

It is an interesting feature of the system, and one that is crucial for the present argument, that we have extensionality of Sum, in that Sum-s are unique, but we do not have extensionality of proper parthood. That is, (3) below is a theorem but (4) is not:

(3) $Sum(xx, y) \land Sum(xx, z) \to y = z$
(4) $C(x) \lor C(y) \to \big((z \ll x \leftrightarrow z \ll y) \to x = y\big)$

It remains to be seen how this relates to emergence. I now turn to that.

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[10] See Cotnoir and Varzi 2021.
[11] This is the system we analyze in detail in Calosi and Giordani 2023b.
[12] Note that REFLEXIVITY ($x < x$) follows.

3. The Account

Given UNRESTRICTED SUM and theorem (3) we can define a total function over the domain of concrete objects that assign to each concrete object its *matter*.[13] More precisely, letting xx be the plurality of proper parts of x, we define the matter of x, $m(x)$ as x if x is simple, and as the *Sum* of the xx if x is composite:

D.7 $S(x) \rightarrow m(x) = x$ SIMPLE-MATTER

D.8 $C(x) \rightarrow m(x) = \iota z\big(Sum(xx, z)\big)$ COMPOSITE-MATTER

Now we can distinguish those objects that are identical to their matter and those that are not. I call the first REDUCIBLE WHOLES, the second IRREDUCIBLE WHOLES:[14]

D.9 $R(x) \equiv x = m(x)$ REDUCIBLE-WHOLE

D.10 $I(x) \equiv x \neq m(x)$ IRREDUCIBLE-WHOLE

Intuitively, this distinction corresponds to the distinction between objects that are nothing over and above their parts, such as e.g., heaps of sands, and objects that are something over and above their parts, e.g., complex structured objects such as table, trees, organisms, statues. The following are immediate consequences:

(5) $S(x) \rightarrow R(x)$

(6) $I(x) \rightarrow C(x)$

None of the converses hold. As a way of illustration, consider the following model, where $\oplus$ is simply "binary *Sum*":[15]
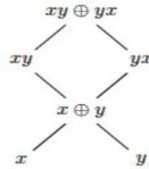

Figure 1: A Model with Reducible and Irreducible Wholes

In the model above $x \oplus y$ is a reducible whole, which is the matter of two irreducible wholes with reducible proper parts, namely xy, and yx, and the matter of a reducible whole with irreducible parts, namely $xy \oplus yx$. It should be clear why the present proposal has a chance to provide a mereology for emergent wholes: it allows for irreducible wholes that are something over and above

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[13] As I pointed out before, I require **P.4** only for the sake of simplicity, but it is unnecessarily strong. All the following arguments require is an existence axiom for *Sum*-s that guarantees that the matter of every entity exists. There are different principles that are (i) are compatible with this requirement, and (ii), weaker than **P.4**.

[14] This mirrors the distinction between *unstructured* and *structured* entities in Calosi and Giordani 2023a.

[15] In Calosi and Giordani 2023a we suggest this is how to account for the infamous case of the composition of a syllable in Aristotle's *Met. Z.*

their proper parts, i.e., their matter. Indeed, I suggest that, faced with cases of emergent wholes (E) we should endorse the following conditional:

(7) $E(x) \rightarrow I(x)$

IRREDUCIBILITY as defined above is a necessary condition for emergence. I want to stay neutral as to whether the converse holds. Indeed, I am more hesitant to subscribe to irreducibility being sufficient for emergence. Perhaps there are other "grounds" for irreducibility. Why should one hold that emergent wholes are irreducible in the precise way I defined them? To answer this question, note that we can extract different broad conditions a mereology for emergent wholes needs to meet from the account of emergence in Wilson 2021. Irreducibility in this precise sense helps meeting this requirement. We saw the first (conjunctive) requirement already:

> *Dependence and Autonomy*: Emergent wholes are somewhat dependent on their parts, but at the same time somehow ontologically autonomous from them.[16]

In Wilson's words:

> Summing up: many considerations, drawn from science, perception, language, our practices of individuation, and introspective experience, provide prima-facie support for thinking that many broadly natural entities are co-temporally materially dependent on micro-configurations of fundamental physical entities, yet are also ontologically and causally autonomous with respect to these underlying micro-configurations (Wilson 2021: 6-7).

> *Compositional Flexibility*: The existence of an emergent whole depends on the existence of its parts but does not depend on the existence of any specific plurality of proper parts.[17] In effect, the emergent whole is usually taken to be capable of surviving (some) changes in mereological structure—see e.g., Wilson 2021: 6.

> *Sortal Properties of Ordinary Objects*: Some emergent wholes, in particular *ordinary objects*, fall under "sortal features" that do not apply to any collection of proper parts of said wholes and are responsible for their persistence conditions.[18]

To quote Wilson again:

> Candidate sortal features for ordinary objects of the varieties at issue here would be feature expressing membership in the category at issue, such as 'being a table' or 'being a statute' (Wilson 2021: 197).

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[16] Wilson (2021) discusses several suggestions to cash out precisely both the *dependence* and the *autonomy* aspects. I will not enter these details here.

[17] It is an interesting question whether this distinction Wilson draws parallels the one in e.g., Simons 1987 between *generic* and *rigid* dependence. My inclination is that both Simons and Wilson are after the same distinction. But the devil is in the details, and I am not sure Wilson would buy the *analysis* of dependence that Simons (1987) puts forward.

[18] Wilson dedicates the entire Chapter 6 to such objects, arguing that they provide an example of Weak Emergence. For Weak Emergence, see Wilson 2021, especially Chapter 3.

As I pointed out already, I want to make a case for the following claim: the mereological system I proposed helps in satisfying all the desiderata above. Consider *dependence*. According to (7), every emergent whole is an irreducible whole, that is, a whole that is distinct from its matter. But note that the *matter* of an irreducible whole is a very *sui-generis* proper part of that whole. In particular it its *only maximal, unsupplemented proper part*. By this I simply mean that every other proper part of the emergent whole is a proper part of its matter, and therefore overlaps its matter. This captures an important sense in which every irreducible whole *depends* on its matter: were we to annihilate its matter, it is unclear that anything would remain of the whole. Note that it is exactly this kind of considerations that are usually taken to be a litmus test for dependence. At the same time, an irreducible whole is *distinct* from its matter. Now, I grant that numerical distinctness is not sufficient for *autonomy*, but I submit, it is at least *necessary*. What about *compositional flexibility*? There is a raging debate over whether mereological sums can undergo mereological changes. But irreducible wholes are exactly those wholes that are *not Sum*-s. Whatever stance one takes on the possibility of Sums of surviving mereological changes, this does not affect the possibility of irreducible wholes to survive such changes. Indeed, the model in Figure 1 shows that different irreducible wholes, such as xy and yx, can have the same matter. Granted, this does not show that the same irreducible whole can have a different matter at different times. Unfortunately, to provide a detailed account of such possibility, one would need to dive deep into the metaphysics of persistence. I cannot do justice to such a project here. I rest content at pointing out that the very distinction between irreducible and reducible wholes provides a leeway to account for both compositional dependence and compositional flexibility. Finally, *sortal properties*. The thought here is that once the distinction between an irreducible whole and its matter is in place, one can simply claim that the relevant sortal property such as e.g., "being a statue" applies to the irreducible whole but not to its matter. The case of the statue is indeed instructive. Let me contrast here the analysis provided by the account I put forward in the paper with another account, that is more familiar in the mereological literature. My contention is that the new account is a better fit with metaphysical emergence.

As we saw in §1 emergent wholes seem to be "something over and above their parts" in virtue of their ontological autonomy. The familiar way of cashing out this proposal in the mereological literature is to endorse a *non-extensional* mereological system, that is, a mereological system that does not have (4) among its axioms or theorems. The system we are investigating is one example. But there are others. Arguably, the most popular one since at least Simons 1987 is the one that endorses Sum_1 as its notion of sum, has **P.1** and **P.2** as its axioms, and replaces **P.3** and P4 with the following respectively:[19]

P.5 $x \ll y \rightarrow \exists z(z < y \land \neg x \circ z)$ Weak Supplementation
P.6 $\exists x(x < xx) \rightarrow \exists z\big(Sum_1(z, xx) \leftrightarrow \varphi(xx)\big)$ Restricted-Composition

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[19] But there are many others. For an introduction see Cotnoir 2013. One needs restricted composition because Weak Supplementation and Transitivity, together with Sum_1, yield (3) as a theorem. See Varzi 2009.

Importantly, in this mereological system Sum_1-s are not unique. That is, (3) is not a theorem of the system. Now, suppose we have a statue, call it *Statue*, that is made out of a lump of clay, call it *Lump*, that has two parts, *Lefty* and *Righty*. According to the more familiar mereological account *Lefty* and *Righty* have two Sum_1-s, namely *Statue* and *Lump*, as in Figure 2 below:
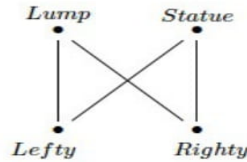

Figure 2: Statue and Lump: Part I

The thought here is that wholes are something over and above their parts in that the existence of proper parts does not determine the identity of the whole. Indeed, different wholes can share the same proper parts. But note that, from a purely mereological perspective, both *Lump* and *Statue* are Sum_1 of *Lefty* and *Righty*. And yet, in the present context, only one of them is an (alleged) emergent whole with a distinguished sortal property such as "being a statue". It seems clear that the mereological structure of the Sum-s cannot account for the difference of the metaphysical status of the wholes with respect to emergence. The mereological system I discussed handles things much differently—and, I contend, better. In the case at hand, there will be only one Sum of *Lefty* and *Righty*, namely *Lump* which is a reducible, hence non-emergent whole. *Lump* is the *matter* of *Statue* which is a distinct, irreducible emergent whole, as per Figure 3:
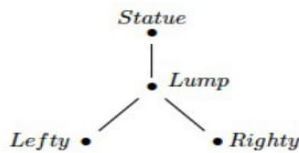

Figure 3: Statue and Lump: Part II

Here, the difference between the composite objects *Lump* and *Statue* is reflected in the mereology so to speak. *Lump* is a Sum, and therefore a reducible object. By contrast *Statue* is not a Sum. It is something over and above its matter—*Lump*—and this is why the emergent sortal property "being a statue" only applies to *it*. This is reason enough to prefer the mereological system I suggested to the one that is more familiar from the literature, at least if one maintains that statues are emergent wholes distinguished by their emergent (sortal) properties.

4. An Application

Beside ordinary objects and artifacts, Wilson (2021) suggests that special-sciences entities might be (at least weakly) emergent. For instance, she writes:

Special-science entities are characterized as having distinctive features, constitutive of the distinctive types under which they fall. A tree, for example, has roots, a trunk, branches, stems, leaves; it obtains nutrients from air, sun, soil, and water through leaves and roots; it reproduces via seeds and may bear fruit; it is deciduous or evergreen; it is hardy in certain climate zones, and so on. On the face of it, such features are not appropriately attributed to even complex configurations of fundamental physical entities; and the same is true for the characteristic features of other special-science entities (Wilson 2021: 4).

To conclude I want to discuss an application of the new mereology for emergence that I suggested to a particular example that combines different special-science entities. The example I have in mind is that of the particular *organism* mentioned in the passage above, a tree.[20] How does the new mereology handle the constitution of an organism such as a tree, where different parts of the tree are arguably themselves weakly emergent entities studied by different special sciences?[21] It is interesting to note that the passage to new special-science level with distinctive weakly emergent wholes is clearly mirrored in the mereological system I proposed. In particular it is mirrored in the passage from a reducible whole to an irreducible one of which the former is the matter. For instance, one starts with *atoms*, studied by *physics*.[22] Sums of atoms provide the matter of other weakly emergent wholes, *molecules*, studied by chemistry. Sums of molecules provide the matter for other weakly emergent wholes, *cells*, studied by *biology*. Finally, sums of cells provide the matter of other weakly emergent wholes, *organisms*, studied in the case of a tree, by *botany*. This is illustrated in Figure 4 below:[23]
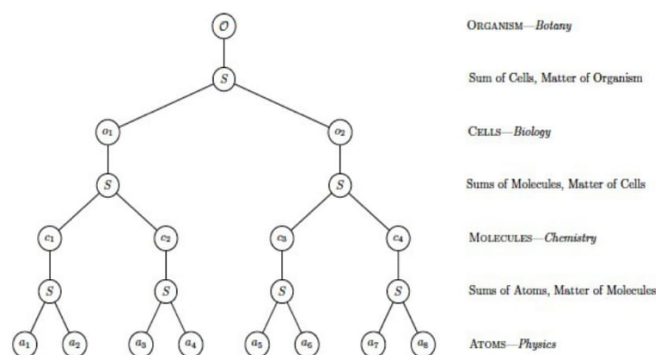

Figure 4: A Tree

[20] See also Calosi and Giordani 2023a.
[21] For a discussion of the relation between emergence, and a layered conception of reality with different levels studied by different special sciences see Wilson 2021: 12 and 24-30.
[22] For a discussion of atomism and emergence see Wilson 2021: 24.
[23] For the sake of clarity, I did not draw all the *Sum*-s.

To sum up. I argued that the possibility of emergence, as characterized in Wilson 2021, poses a threat to various reductive accounts of mereological structure. I then proposed a new account that seems to fit well with various intimations coming from the metaphysics of emergence, as applied to paradigmatic cases of emergent wholes. I admit this is just a first rung of a more thorough investigation of the mereological ladder of such emergent wholes. The hope is that this rung stands on solid ground.[24]

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